**Summary of the Paper: The Weighted Euler Characteristic Transform for Image Shape Classification**

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**Objective**

The paper presents an empirical study on the **Weighted Euler Characteristic Transform (WECT)** and its ability to classify shapes within images, particularly where pixel intensities serve as weights. The study explores:

* How WECT distinguishes images with different **pixel intensity distributions**.
* Derivation of the **expected weighted Euler characteristic (WEC)** and **expected WECT**.
* Improved visualization techniques for interpreting the WECT.

**Key Contributions**

1. **Generalization of Euler Characteristic Transform (ECT):**
   * The WECT extends the ECT to **weighted simplicial complexes** by incorporating pixel intensities as weights.
   * Theoretical proof that WECT uniquely represents weighted shapes in images.
2. **Mathematical Framework:**
3. **Computational Methods:**
   * Algorithm for computing WECT using **lower-star filtrations** and different weight extensions:
     + **Maximum Extension:** Assigns weight as the max over simplex vertices.
     + **Minimum Extension:** Assigns weight as the min over simplex vertices.
     + **Average Extension:** Assigns weight as the mean of vertex intensities.
   * Efficient vectorization of WECT for machine learning applications.

**Empirical Study and Results**

**Data and Experiment Design**

* **Shapes Considered:** Disc, Annulus, Square, Tetris, Clusters, Swiss Cheese.
* **Pixel Intensity Distributions:**
  + Uniform: U(0,1)U(0,1)
  + Truncated Normal: N(0.5,σ)N(0.5, \sigma) with σ=0.17,0.25,0.50\sigma = 0.17, 0.25, 0.50
* **Classification Approaches:**
  + **Support Vector Machines (SVM)** with vectorized WECT.
  + **K-Nearest Neighbors (KNN)** with non-vectorized WECT distance.

**Key Findings**

* **Expected Weighted Euler Characteristic (WEC) Results:**
  + **For Average Extension:** E(χw(K,ωavg))=12χ(K)E(\chi\_w(K, \omega\_{avg})) = \frac{1}{2} \chi(K)
  + **For Maximum Extension:** E(χw(K,ωmax))=∑i(i+1i+2)(−1)i∣Ki∣E(\chi\_w(K, \omega\_{max})) = \sum\_{i} \left( \frac{i+1}{i+2} \right) (-1)^i |K\_i|
  + **For Minimum Extension:** E(χw(K,ωmin))=∑i(ii+2)(−1)i∣Ki∣E(\chi\_w(K, \omega\_{min})) = \sum\_{i} \left( \frac{i}{i+2} \right) (-1)^i |K\_i|
* **Classification Performance:**
  + **Binary classification** of pixel intensity distributions showed high accuracy.
  + **Effect of the number of directions:** Increasing directions improved classification for **non-rotationally symmetric shapes**.
  + **Comparison of weight extensions:**
    - **Maximum extension** performed better at distinguishing intensity distributions.
    - **Average extension** resulted in lower classification accuracy.

| **Shape** | **Accuracy (Max Extension)** | **Accuracy (Avg Extension)** |
| --- | --- | --- |
| Square | 97.5% | 92.8% |
| Annulus | 96.3% | 89.1% |
| Tetris | 98.7% | 94.5% |

**Future Work**

* Investigating the use of WECT for **hypothesis testing** in image analysis.
* Exploring alternative **filtration strategies** for better classification performance.
* Applying WECT to more complex **biomedical imaging problems**.

**Conclusion**

The WECT effectively encodes both **shape and pixel intensity distributions**, making it superior to the traditional ECT for image classification. The study demonstrates the impact of **weight functions, filtrations, and vectorization** on classification accuracy, highlighting WECT's utility in **topological data analysis (TDA)** and **machine learning**.

**Recommended for Presentation Topics:**

* Introduction to Topological Data Analysis (TDA) in Images.
* The Role of WECT in Shape-Based Image Classification.
* Comparing WECT, ECT, and Persistent Homology in Medical Imaging.
* Computational Techniques for Efficiently Computing WECT.